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Mass Spectra of Supersymmetric Yang-Mills Theories in $1+1$ Dimensions

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Abstract

Physical mass spectra of supersymmetric Yang-Mills theories in $1+1$ dimensions are evaluated in the light-cone gauge with a compact spatial dimension. The supercharges are constructed and the infrared regularization is unambiguously prescribed for supercharges, instead of the light-cone Hamiltonian. This provides a manifestly supersymmetric infrared regularization for the discretized light-cone approach. By an exact diagonalization of the supercharge matrix between up to several hundred color singlet bound states, we find a rapidly increasing density of states as mass increases.

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1. Introduction

Supersymmetric theories offer promising models for the unified theory. Both as a model for grand unified theories and as a low energy effective theory for superstring, the dynamics of supersymmetric Yang-Mills gauge theories is a fascinating subject. The nonperturbative aspects of supersymmetric theories are crucial to understand fundamental aspects of such theories, especially the supersymmetry breaking.

One of the most popular models for the supersymmetry breaking is currently to assume the gaugino bilinear condensation in the supersymmetric Yang-Mills gauge theories [1]. Although the condensation itself may not break supersymmetry in the supersymmetric gauge theories, it will give rise to the supersymmetry breaking if embedded in supergravity [2]. Since the fermion bilinear condensation is implied by the chiral symmetry breaking in QCD, one can expect a similar nonperturbative effects in supersymmetric Yang-Mills gauge theories. Moreover, recent progress in understanding duality in supersymmetric Yang-Mills gauge theories opened up a rich arena for studying the nonperturbative effects in supersymmetric gauge theories [3].

It has been quite fruitful to study Yang-Mills gauge theories in $1 + 1$ dimensions instead of studying directly the four dimensional counterpart. In $1 + 1$ dimensions, Yang-Mills gauge field itself has no dynamical degree of freedom as a field theory, but gives rise to a confining potential for colored particles [4]. Many aspects of color singlet bound states can be explored by solving the theory in the large N limit [5]. Unfortunately the supersymmetric gauge multiplet contains genuine dynamical degree of freedom in the adjoint representation of the gauge group contrary to ordinary Yang-Mills gauge theory [6]. Therefore one cannot obtain a simple closed form for the color singlet bound states even in the large N limit.

There has been progress in studying the dynamics of matter fields in the adjoint representation in ordinary Yang-Mills gauge theories [7]. They have used the light-cone quantization and compactified the spatial dimension to give discrete momenta. In this discretized light-cone quantization approach, one can diagonalize the mass matrix for finite number of light-cone momenta and can hope to obtain the infinite volume limit eventually [8], [9]. The Yang-Mills gauge theory with only the adjoint matter fermion is used to propose a kind of supersymmetry which is valid only at a particular value of a parameter and is different from the usual linearly realized supersymmetry [10]. More recently, gauge theories in $1 + 1$ dimensions with matter in adjoint representations was studied focusing attention on zero modes [11]. The

zero modes are generally important in revealing nontrivial vacuum structures such as the vacuum condensate [12].

In spite of these investigations of Yang-Mills gauge theories with adjoint scalar and spinor matter fields, there are two points which necessitate a new analysis of physical spectra in the case of supersymmetric gauge theories. The first point is that the coexistence of spinor and scalar gives rise to a large number of new “mixed” physical states, partly consisting of spinors and partly of scalars as constituents. The second point is the presence of a specific amount of the Yukawa interaction which is a distinguishing feature of the supersymmetric Yang-Mills theory [6].

The purpose of our paper is to study the supersymmetric Yang-Mills gauge theories in $1 + 1$ dimensions through the discretized light-cone quantization. We construct the supercharge explicitly and specify an infrared regularization for supercharge by means of the discretized version of the principal value prescription. By using the supercharge, we succeed in overcoming ambiguities in prescribing the infrared regularization for the light-cone Hamiltonian. As a result, the regularization preserves the supersymmetry algebra manifestly. For light-cone momenta up to 8 units of the smallest momentum, we find several hundred color singlet bound states of bosons and the same number of fermions. We exactly diagonalize the supercharge instead of the Hamiltonian to obtain masses, degeneracies, and the average number of constituents in these bound states. We observe that the density of the bound states as a function of their masses tends to converge in the large volume limit. It is consistent with the rapidly increasing density of states suggested by the closed string interpretation. Since we preserve supersymmetry at each stage of our study, we naturally obtain exact correspondence between bosonic and fermionic color singlet bound states. Although we postpone studying the issue of zero modes, our results in the present approximation suggest that supersymmetry is not broken in this supersymmetric Yang-Mills gauge theory. It is an interesting future problem to see if our supersymmetric theory can offer a model for gaugino condensation. For that purpose, one should study the zero mode in this theory. However, the present investigation is focused on physical mass spectra as a first step to understand the dynamics of the supersymmetric Yang-Mills theories.

Before writing up our paper, we have received a preprint, where the same theory has been studied by means of the Makeenko-Migdal loop equations [13]. With certain assumptions, the author gave an interesting solution and also argued for the nonvanishing Witten index. Although his method is worth exploring, it may not be suitable to obtain physical quantities such as mass spectra. In this respect, our

methods are complimentary to his, and our conclusions are consistent with each other.

In sect. 2, supersymmetric Yang-Mills gauge theories in $1 + 1$ dimensions are quantized and supercharges are defined. In sect. 3, the compact spatial dimension is introduced in the light-cone quantization and the supercharges are discretized. The result of our exact diagonalization of supercharge is presented and discussed in sect. 4. Superfields and supertransformations are summarized in Appendix A. Truncation of the bound state equation to the two constituents subspace is given in Appendix B. Explicit mass matrices with mass terms for adjoint scalar and spinor is given in Appendix C.

2. SUSY Yang-Mills Theories in $1 + 1$ Dimensions

In two-dimensions, the gauge field A^μ is contained in a supersymmetric multiplet consisting of a Majorana fermion Ψ and a scalar ϕ in the adjoint representation of the gauge group together with gauge field itself [6]. Therefore our field content is different from that in ref [10]. After choosing the Wess-Zumino gauge, we have an action

$$S = \int d^2x \operatorname{tr} \left[-\frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} D_\mu \phi D^\mu \phi + i\bar{\Psi} \gamma^\mu D_\mu \Psi - 2ig\phi \bar{\Psi} \gamma_5 \Psi \right], \quad (1)$$

where A_μ , ϕ , Ψ , and $\bar{\Psi} = \Psi^T \gamma^0$ are traceless $N \times N$ hermitian matrix for $U(N)$ ($SU(N)$) gauge group, g is the gauge coupling constant, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]$ and D_μ is the usual covariant derivative

$$D_\mu \phi = \partial_\mu \phi + i[A_\mu, \phi], \quad D_\mu \Psi = \partial_\mu \Psi + i[A_\mu, \Psi]. \quad (2)$$

The supersymmetry dictates the presence of the Yukawa type interaction between the adjoint spinor and scalar fields with the strength of the gauge coupling. The supersymmetric Yang-Mills gauge theory in two-dimensions can be obtained by a dimensional reduction from the supersymmetric Yang-Mills gauge theory in three dimensions. The adjoint scalar field can be understood as the component of the gauge field in the compactified dimension and the Yukawa coupling is nothing but the gauge interaction in this compactified extra dimension.

In the Wess-Zumino gauge, the remaining invariances of the action are the usual gauge invariance and a supertransformation which is obtained by combining the

supertransformation and the compensating gauge transformation in the superfield formalism as summarized in the Appendix A. We denote this modified supertransformation as $\tilde{\delta}_{super}$ which is given in terms of component fields as ($\epsilon^{01} = -\epsilon_{01} = 1$)

$$\begin{aligned}\tilde{\delta}_{super}A_\mu &= ig\bar{\epsilon}\gamma_5\gamma_\mu\sqrt{2}\Psi, \quad \tilde{\delta}_{super}\phi = -\bar{\epsilon}\sqrt{2}\Psi, \\ \tilde{\delta}_{super}\Psi &= -\frac{1}{2\sqrt{2}g}\epsilon^{\mu\nu}F_{\mu\nu} + \frac{i}{\sqrt{2}}\gamma^\mu\epsilon D_\mu\phi.\end{aligned}\quad (3)$$

The corresponding spinor supercurrent j^μ is given by

$$\bar{\epsilon}j^\mu = \text{tr} \left[-\sqrt{2}\bar{\epsilon}\Psi D^\mu\phi + i\frac{1}{\sqrt{2}g}\epsilon^{\nu\lambda}F_{\nu\lambda}\bar{\epsilon}\gamma^\mu\Psi + \sqrt{2}\bar{\epsilon}\gamma_5\Psi\epsilon^{\mu\nu}D_\nu\phi \right]. \quad (4)$$

We introduce the light-cone coordinates where the line element ds^2 is given by

$$x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^1), \quad ds^2 = (dx^0)^2 - (dx^1)^2 = 2dx^+dx^-. \quad (5)$$

We decompose the spinor and use gamma matrices

$$\Psi_{ij} = 2^{-1/4}(\psi_{ij}, \chi_{ij})^T, \quad \gamma^0 = \sigma_2, \quad \gamma^1 = i\sigma_1, \quad \gamma_5 = \gamma^0\gamma^1 = \sigma_3. \quad (6)$$

Taking the light-cone gauge $A_- = A^+ = 0$ and x^+ as time, we find the action

$$\begin{aligned}S &= \int dx^+dx^- \text{tr} \left[\partial_+\phi\partial_-\phi + i\psi\partial_+\psi + i\chi\partial_-\chi \right. \\ &\quad \left. + \frac{1}{2g^2}(\partial_-A_+)^2 + A_+J^+ + \sqrt{2}g\phi\{\psi, \chi\} \right],\end{aligned}\quad (7)$$

where the current J^+ receives contributions from the scalar J_ϕ^+ and the spinor J_ψ^+

$$J^+ = J_\phi^+ + J_\psi^+, \quad J_\phi^+ = i[\phi, \partial_-\phi], \quad J_\psi^+ = 2\psi\psi. \quad (8)$$

We do not need Faddeev-Popov ghosts in this gauge. Since the action contains no time derivative for the gauge potential A_+ and the left-moving fermion χ , they can be eliminated by means of constraints obtained as their Euler-Lagrange equations

$$i\sqrt{2}\partial_-\chi - g[\phi, \psi] = 0, \quad \partial_-^2\bar{A}_+ - g^2J^+ = 0. \quad (9)$$

where \bar{A}_+ is the non-zero mode of A_+ . The zero mode of A_+ plays the role of a Lagrange multiplier which provides a constraint

$$\int dx^- J^+ = 0. \quad (10)$$

This constraint will give a restriction for physical states in quantum theory. After eliminating the fields A_+ and χ , we find that the action becomes

$$S = \int dx^+ dx^- \text{tr} \left[\partial_+ \phi \partial_- \phi + i\psi \partial_+ \psi + \frac{g^2}{2} J^+ \frac{1}{\partial_-^2} J^+ - \frac{1}{2} i g^2 [\phi, \psi] \frac{1}{\partial_-} [\phi, \psi] \right]. \quad (11)$$

Let us note that the constraints give rise to non-local terms in the action.

By the Noether procedure, we construct the energy momentum tensor $T^{\mu\nu}$, and light-cone momentum and energy $P^\pm = \int dx^- T^{+\pm}$ on a constant light-cone time

$$P^+ = \int dx^- \text{tr} \left[(\partial_- \phi)^2 + i\psi \partial_- \psi \right], \quad (12)$$

$$P^- = \int dx^- \text{tr} \left[-\frac{g^2}{2} J^+ \frac{1}{\partial_-^2} J^+ + \frac{i}{2} g^2 [\phi, \psi] \frac{1}{\partial_-} [\phi, \psi] \right]. \quad (13)$$

The supercharges Q_1 and Q_2 are defined as integrals of the upper and lower components of the spinor supercurrent $j^\mu = (j_1^\mu, j_2^\mu)$ in eq.(4)

$$Q_1 \equiv \int dx^- j_1^+ = 2^{1/4} \int dx^- \text{tr} [\phi \partial_- \psi - \psi \partial_- \phi], \quad (14)$$

$$\begin{aligned} Q_2 &\equiv \int dx^- j_2^+ = 2^{3/4} g \int dx^- \text{tr} \left[J^+ \frac{1}{\partial_-} \psi \right] \\ &= 2^{3/4} g \int dx^- \text{tr} \left\{ (i[\phi, \partial_- \phi] + 2\psi \psi) \frac{1}{\partial_-} \psi \right\}. \end{aligned} \quad (15)$$

Using the conjugate momenta $\pi_\phi = \partial \mathcal{L} / \partial(\partial_+ \phi) = \partial_- \phi$ for adjoint scalar field ϕ_{ij} and $\pi_\psi = \partial \mathcal{L} / \partial(\partial_+ \psi) = i\psi$ for adjoint spinor field ψ_{ij} , the canonical (anti)commutation relation are given at equal light-cone time $x^+ = y^+$ by

$$[\phi_{ij}(x), \partial_- \phi_{kl}(y)] = i \{ \psi_{ij}(x), \psi_{kl}(y) \} = \frac{1}{2} i \delta(x^- - y^-) \delta_{il} \delta_{jk}. \quad (16)$$

We expand the fields in modes with momentum k^+ at light-cone time $x^+ = 0$

$$\phi_{ij}(x^-, 0) = \frac{1}{\sqrt{2\pi}} \int_0^\infty \frac{dk^+}{\sqrt{2k^+}} \left(a_{ij}(k^+) e^{-ik^+ x^-} + a_{ji}^\dagger(k^+) e^{ik^+ x^-} \right), \quad (17)$$

$$\psi_{ij}(x^-, 0) = \frac{1}{2\sqrt{\pi}} \int_0^\infty dk^+ \left(b_{ij}(k^+) e^{-ik^+ x^-} + b_{ji}^\dagger(k^+) e^{ik^+ x^-} \right). \quad (18)$$

The canonical (anti-)commutation relations (16) are satisfied by

$$[a_{ij}(k^+), a_{lk}^\dagger(\tilde{k}^+)] = \{b_{ij}(k^+), b_{lk}^\dagger(\tilde{k}^+)\} = \delta(k^+ - \tilde{k}^+) \delta_{il} \delta_{jk}. \quad (19)$$

In nonsupersymmetric theories, one can define finite Hamiltonian operators only after discarding the usually divergent vacuum energies [7]. However, we should not discard any vacuum energies in supersymmetric theories, since vacuum energies have an absolute meaning in supersymmetric theories as an indicator of supersymmetry breaking. In fact we will not need to discard the vacuum energies by hand, provided we exercise care with respect to ordering of operators.

One can obtain the light-cone momentum P^+ in terms of oscillators

$$P^+ = \int_0^\infty dk k \left\{ a_{ij}^\dagger(k) a_{ij}(k) + b_{ij}^\dagger(k) b_{ij}(k) \right\}, \quad (20)$$

where we dropped the superscript $+$ on k^+ for brevity, and henceforth we do so.

The light-cone Hamiltonian P^- can be divided into two parts: the current-current interaction term P_{JJ}^- and the Yukawa coupling term P_{Yukawa}^-

$$P^- = P_{JJ}^- + P_{Yukawa}^-. \quad (21)$$

Let us introduce the momentum representation of the current J^+

$$\tilde{J}^+(k) = \frac{1}{\sqrt{2\pi}} \int dx^- J^+(x^-) \exp(-ikx^-). \quad (22)$$

Substituting the mode expansions (17) and (18), we obtain for $-k < 0$

$$\begin{aligned} \tilde{J}_{ij}^+(-k) &= \frac{1}{2\sqrt{2\pi}} \int_0^\infty dp \frac{2p+k}{\sqrt{p(p+k)}} \left[a_{ki}^\dagger(p) a_{kj}(k+p) - a_{jk}^\dagger(p) a_{ik}(k+p) \right] \\ &+ \frac{1}{2\sqrt{2\pi}} \int_0^k dp \frac{k-2p}{\sqrt{p(k-p)}} a_{ik}(p) a_{kj}(k-p) \\ &+ \frac{1}{\sqrt{2\pi}} \int_0^\infty dp \left[b_{ki}^\dagger(p) b_{kj}(k+p) - b_{jk}^\dagger(p) b_{ik}(k+p) \right] + \frac{1}{\sqrt{2\pi}} \int_0^k dp b_{ik}(p) b_{kj}(k-p). \end{aligned} \quad (23)$$

Note that $\tilde{J}_{ij}^+(k) = [\tilde{J}_{ji}^+(-k)]^\dagger$. There are no (divergent) c-number terms in $\tilde{J}^+(k)$ since supersymmetry requires for the bosonic and fermionic c-number contributions to cancel each other. Therefore $\tilde{J}^+(k)$ is just the same as the normal ordered product : $\tilde{J}^+(k)$: . The current-current interaction term is given by

$$P_{JJ}^- = \frac{g^2}{2} \int_{-\infty}^\infty \frac{dk}{k^2} \tilde{J}_{ij}^+(k) \tilde{J}_{ji}^+(-k) = \frac{g^2}{2} \int_0^\infty \frac{dk}{k^2} \left\{ \tilde{J}_{ij}^+(k), \tilde{J}_{ji}^+(-k) \right\}. \quad (24)$$

The source for χ is given in the momentum space for $-k < 0$

$$[\widetilde{\phi, \psi}]_{ij}(-k) = \frac{1}{2\sqrt{2\pi}} \int_0^\infty dp \left\{ \frac{1}{p} \left[a_{ki}^\dagger(p) b_{kj}(k+p) - a_{jk}^\dagger(p) b_{ik}(k+p) \right] \right.$$

$$\begin{aligned}
& + \frac{1}{k+p} \left[a_{ik}(k+p) b_{jk}^\dagger(p) - a_{kj}(k+p) b_{ki}^\dagger(p) \right] \Big\} \\
& + \frac{1}{2\sqrt{2\pi}} \int_0^k dp \frac{1}{\sqrt{p}} [a_{ik}(p) b_{kj}(k-p) - a_{kj}(p) b_{ik}(k-p)],
\end{aligned} \tag{25}$$

Note that $[\widetilde{\phi}, \widetilde{\psi}]_{ij}(k) = -[\widetilde{\phi}, \widetilde{\psi}]_{ji}(-k)^\dagger$. The Yukawa coupling term is given by

$$P_{Yukawa}^- = -\frac{g^2}{2} \int_0^\infty \frac{dk}{k} \left[[\widetilde{\phi}, \widetilde{\psi}]_{ij}(k), [\widetilde{\phi}, \widetilde{\psi}]_{ji}(-k) \right]. \tag{26}$$

When we bring the Hamiltonian into a normal ordered form, we find that the (divergent) c-number vacuum energies cancel between bosons and fermions. Moreover, the only additional term P_{quad}^- compared to normal ordered Hamiltonian : P^- : is quadratic and is symmetric between scalar and spinor oscillators

$$P^- = P_{quad}^- + : P^- : \tag{27}$$

$$P_{quad}^- = \frac{g^2}{4\pi} \int_0^\infty \frac{dk}{k} C(k) \left(a_{ij}^\dagger(k) a_{lm}(k) + b_{ij}^\dagger(k) b_{lm}(k) \right) (N\delta_{il}\delta_{jm} - \delta_{ij}\delta_{lm}) \tag{28}$$

$$C(k) \equiv \int_0^k dp \left(\frac{4k}{p^2} + \frac{1}{p} \right) = \int_0^k dp \frac{(k+p)^2}{p(k-p)^2} \tag{29}$$

The supercharge is given in terms of these operators as

$$Q_1 = i2^{1/4} \int_0^\infty dk \sqrt{k} \left[a_{ij}(k) b_{ij}^\dagger(k) - a_{ij}^\dagger(k) b_{ij}(k) \right], \tag{30}$$

$$Q_2 = -i2^{1/4} g \int_0^\infty \frac{dk}{k} \left[b_{ij}^\dagger(k) \tilde{J}_{ij}(-k) - \left(\tilde{J}_{ij}(-k) \right)^\dagger b_{ij}(k) \right]. \tag{31}$$

By taking (anti-)commutators with spinor ψ and scalar ϕ fields, we can confirm that these supercharge operators generate supertransformations in the light-cone gauge as given in (15) and (16).

Our next task is to determine physical states whose mass spectra will be calculated later. The light-cone vacuum is the Fock vacuum defined by

$$a_{ij}(k) |0\rangle = 0, \quad b_{ij}(k) |0\rangle = 0. \tag{32}$$

satisfying $P^\pm |0\rangle = 0$. Fock states are given by acting creation operators $a_{ij}^\dagger(k), b_{ij}^\dagger(k)$ and their linear combinations on $|0\rangle$. In leading order in the $1/N$ expansion, physical states are given by gauge singlet states with single trace of creation operators $\text{tr} [\mathcal{O}(k_1) \cdots \mathcal{O}(k_m)] |0\rangle / N^{m/2} \sqrt{s}$ with $\mathcal{O}(k)$ representing $a^\dagger(k)$ or $b^\dagger(k)$, and $N^{-m/2}$ the normalization factor and s a symmetry factor.

The mass spectrum is obtained by solving the eigenvalue problem

$$2P^+P^-|\Phi\rangle = M^2|\Phi\rangle. \quad (33)$$

It is almost impossible to solve the eigenvalue problem analytically because we must diagonalize an infinite dimensional matrix. Therefore we will resort to a discretized approximation in the next section. Truncation to two constituent subspace yields a closed bound state equation similar to the 'tHooft equation [4] as described in Appendix B.

3. Discretized Light-Cone Quantization of Superchge

In order to prescribe the infrared regularization precisely and to evaluate the mass spectrum in spaces with finite number of physical states, we compactify spatial direction x^- to form a circle with radius $2L$ by identifying $x^- = 0$ and $x^- = 2L$. In order to preserve supersymmetry, we need to impose the same boundary condition on scalars ϕ_{ij} and spinors ψ_{ij} . It is in general necessary to choose periodic boundary conditions on bosonic field and to retain zero modes, if one wishes to take into account the possibility of vacuum condensate or spontaneous symmetry breaking [12]. Since we are primarily interested in physical mass spectrum, we neglect the zero modes in the present work. We shall choose periodic boundary conditions for both scalars ϕ_{ij} and spinors ψ_{ij} , leaving the problem of zero modes for a further study

$$\phi_{ij}(x^-) = \phi_{ij}(x^- + 2L), \quad \psi_{ij}(x^-) = \psi_{ij}(x^- + 2L). \quad (1)$$

The allowed momenta become discrete and the momentum integral is replaced by a summation,

$$k_n^+ = \frac{\pi}{L}n, \quad n = 1, 2, 3, \dots, \quad \int_0^\infty dk^+ \rightarrow \frac{\pi}{L} \sum_{n=1}^\infty. \quad (2)$$

Then mode expansions (17) and (18) for ϕ_{ij} and ψ_{ij} become discretized

$$\phi_{ij} = \frac{1}{\sqrt{4\pi}} \sum_{n=1}^\infty \frac{1}{\sqrt{n}} \left[A_{ij}(n) e^{-i\pi n x^- / L} + A_{ji}^\dagger(n) e^{i\pi n x^- / L} \right], \quad (3)$$

$$\psi_{ij} = \frac{1}{\sqrt{4L}} \sum_{n=1}^\infty \left[B_{ij}(n) e^{-i\pi n x^- / L} + B_{ji}^\dagger(n) e^{i\pi n x^- / L} \right], \quad (4)$$

$$A_{ij}(n) = \sqrt{\pi/L} a_{ij}(k^+ = \pi n/L), \quad B_{ij}(n) = \sqrt{\pi/L} b_{ij}(k^+ = \pi n/L), \quad (5)$$

$$[A_{ij}(n), A_{lk}^\dagger(n')] = \{B_{ij}(n), B_{lk}^\dagger(n')\} = \delta_{nn'} \delta_{il} \delta_{jk}. \quad (6)$$

Let us define the supercharge in this discretized light-cone quantization. The first supercharge Q_1 in eq.(30) in this compactified space is given by

$$Q_1 = 2^{1/4} i \sqrt{\frac{\pi}{L}} \sum_{n=1}^{\infty} \sqrt{n} [A_{ij}(n) B_{ij}^\dagger(n) - A_{ij}^\dagger(n) B_{ij}(n)]. \quad (7)$$

Since the elimination of gauge field A_+ introduces a singular factor $1/\partial_-$ in supercharge Q_2 in eq.(31), we need to specify an infrared regularization for this factor. Following the procedure of 'tHooft [4], we employ the principal value prescription for the supercharge. Namely we simply drop the zero momentum mode

$$\begin{aligned} Q_2 &= 2^{1/4} (-i) g \sqrt{\frac{L}{\pi}} \sum_{m=1}^{\infty} \frac{1}{m} [B_{ij}^\dagger(m) \tilde{J}_{ij}(-m) - (\tilde{J}_{ij}(-m))^\dagger B_{ij}(m)] \\ &= -i \frac{2^{-1/4} g}{\pi} \sqrt{L} \left(\sum_{l,n=1}^{\infty} \frac{l+2n}{2l\sqrt{n(l+n)}} \left[(A^\dagger(n) B^\dagger(l) - B^\dagger(l) A^\dagger(n))_{ij} A_{ij}(l+n) \right. \right. \\ &\quad \left. \left. - A_{ij}^\dagger(l+n) (A(n) B(l) - B(l) A(n))_{ij} \right] \right. \\ &\quad \left. + \sum_{l=3}^{\infty} \sum_{n=1}^{l-1} \frac{l-2n}{2l\sqrt{n(l-n)}} \left[B_{ij}^\dagger(l) (A(n) A(l-n))_{ij} - (A^\dagger(n) A^\dagger(l-n))_{ij} B_{ij}(l) \right] \right. \\ &\quad \left. - \sum_{l,n=1}^{\infty} \left(\frac{1}{l} + \frac{1}{n} \right) \left[(B^\dagger(n) B^\dagger(l))_{ij} B_{ij}(l+n) + B_{ij}^\dagger(l+n) (B(n) B(l))_{ij} \right] \right. \\ &\quad \left. + \sum_{l=2}^{\infty} \sum_{n=1}^{l-1} \frac{1}{l} \left[B_{ij}^\dagger(l) (B(n) B(l-n))_{ij} + (B^\dagger(n) B^\dagger(l-n))_{ij} B_{ij}(l) \right] \right). \quad (8) \end{aligned}$$

The supersymmetry algebra requires a relation between supercharges and the light-cone momentum P^+ and the Hamiltonian P^- operators

$$\{Q_1, Q_1\} = 2\sqrt{2}P^+, \quad (9)$$

$$\{Q_2, Q_2\} = 2\sqrt{2}P^-, \quad (10)$$

$$\{Q_1, Q_2\} = 0, \quad (11)$$

in our choice of spinor notations (6). Infrared regularizations of P^+ and P^- have to be done consistently with the supersymmetry algebra. It is actually difficult to guess the correct infrared regularization for the Hamiltonian unless we start from the

supercharge. The Hamiltonian P^- can be defined by just squaring the supercharge Q_2 . Then the above principal value prescription for the supercharge Q_2 specifies uniquely the prescription for the Hamiltonian. In this way we can check that the supersymmetry algebra holds in our formulation of the discretized light-cone quantization.

Physical states take the following form

$$\frac{1}{N^{m/2}\sqrt{s}} \text{tr} [\mathcal{O}(n_1) \cdots \mathcal{O}(n_m)] |0\rangle, \quad m > 1, \quad (12)$$

where \mathcal{O} represents A^\dagger or B^\dagger . The symmetry factor s is the number of possible permutations of constituents which give the same state [7]. Note that we should consider only states with two or more constituents $m > 1$ since we should discard singlet to the leading order of the $1/N$ expansion of $U(N)$ gauge theory. It is also absent in the case of $SU(N)$ gauge theory anyway. All these states satisfy the physical state condition coming from the constraint (10)

$$\tilde{J}_{ij}^+(0) |\Phi\rangle = 0. \quad (13)$$

Here we note that there are both bosonic and fermionic oscillators in our supersymmetric theory. This fact gives rise to much larger number of new physical states compared to the purely fermionic or bosonic adjoint matter case.

Since P^+ commutes with other operators, we work on a subspace with a definite value of the light-cone momentum P^+

$$P^+ = \frac{L}{\pi} K, \quad K = 1, 2, \cdots \quad (14)$$

$$K = \sum_{n=1}^{\infty} n \left\{ A_{ij}^\dagger(n) A_{ij}(n) + B_{ij}^\dagger(n) B_{ij}(n) \right\}. \quad (15)$$

For the state defined in (12), $K = \sum_{i=1}^m n_i$. Therefore the number of physical states is finite for a given K . So long as K is finite, we can consider finite dimensional physical state space to diagonalize the mass matrix. The parameter K plays the role of the infrared cut-off. The infinite volume limit $L \rightarrow \infty$ is achieved by taking the limit $K \rightarrow \infty$ with finite physical values of P^+ fixed. As usual in the discretized light-cone approach, we shall evaluate mass spectra for finite K corresponding to a finite spatial box and try to evaluate the asymptotic behavior $K \rightarrow \infty$.

The supersymmetry algebra (10) implies that the diagonalization of the supercharge Q_2 gives the desired mass spectrum. Let us consider the subspace for fixed

light-cone momenta P^+ , and denote

$$Q_1 = \begin{bmatrix} 0 & A^\dagger \\ A & 0 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 0 & B^\dagger \\ B & 0 \end{bmatrix}, \quad (16)$$

where the first half of the rows and columns correspond to the bosonic color singlet bound states and the second half to the fermionic states. The mass matrix is

$$M^2 \equiv 2P^+P^- = \frac{\sqrt{2}\pi K}{L} \begin{bmatrix} B^\dagger B & 0 \\ 0 & BB^\dagger \end{bmatrix}. \quad (17)$$

The diagonalization of the positive definite matrix $B^\dagger B$ gives the mass eigenstates of bosonic color singlet bound states and the other positive definite matrix BB^\dagger gives fermionic ones. There exist two unitary matrices U and V such that

$$U^{-1}(B^\dagger B)U = V^{-1}(BB^\dagger)V = D, \quad U^\dagger U = V^\dagger V = 1. \quad (18)$$

where the matrix D is positive diagonal. Let us emphasize that the positive definiteness of mass squared matrices $B^\dagger B$ and BB^\dagger is a direct consequence of regularizing the supercharge Q instead of P^- .

The relation (9) shows that the matrix A is unitary apart from a scale factor

$$\tilde{A}\tilde{A}^\dagger = 1, \quad \tilde{A} \equiv 2^{\frac{1}{4}} \sqrt{\frac{\pi K}{L}} A, \quad (19)$$

The anticommutation relation between two supercharges (11) gives

$$B^\dagger = -\tilde{A}^\dagger B \tilde{A}^\dagger, \quad (20)$$

Therefore we find that the matrix \tilde{A} is precisely the matrix which maps the mass eigenstates of bosonic bound states and fermionic ones.

$$V = \tilde{A}U. \quad (21)$$

In the rest of this section, we consider adding (supersymmetry-breaking) mass terms m_b for the adjoint scalar field and m_f for spinor field to explore supersymmetry breaking and to help treat the zero modes more precisely

$$S_{massive} = S + \int d^2x \operatorname{tr} \left[-\frac{1}{2} m_b^2 \phi^2 - m_f \bar{\Psi} \Psi \right], \quad (22)$$

where S is the massless action given in eq.(1). In the light-cone gauge $A_- = 0$, the action reduces to mass terms added to the massless action S in eq.(7)

$$S_{massive} = S + \int dx^+ dx^- \operatorname{tr} \left[-\frac{1}{2} m_b^2 \phi^2 - i\sqrt{2} m_f \chi \psi \right]. \quad (23)$$

The Euler-Lagrange equation for the auxiliary field χ is modified from eq.(9)

$$i\sqrt{2}\partial_-\chi - g[\phi, \psi] - im_f\psi = 0. \quad (24)$$

Elimination of A_+ and χ gives the action with S in eq.(11) and mass terms

$$S_{massive} = S + \int dx^+ dx^- \text{tr} \left[-\frac{1}{2}m_b^2\phi^2 + \frac{i}{2}m_f^2\psi\frac{1}{\partial_-}\psi + m_fg\psi\frac{1}{\partial_-}[\phi, \psi] \right]. \quad (25)$$

The momentum $P_{massive}^+$ is the same as eq.(12) and the Hamiltonian $P_{massive}^-$ has mass terms in addition to the massless P^- in eq.(13)

$$P_{massive}^- = P^- + P_{m,quad}^- + P_{m,cubic}^- \quad (26)$$

$$P_{m,quad}^- = \int dx^- \text{Tr} \left[\frac{1}{2}m_b^2\phi^2 - \frac{i}{2}m_f^2\psi\frac{1}{\partial_-}\psi \right], \quad (27)$$

$$P_{m,cubic}^- = -m_fg \int dx^- \text{Tr} \left(\psi\frac{1}{\partial_-}[\phi, \psi] \right). \quad (28)$$

The final result for the additional terms in the Hamiltonian is given in the discretized light-cone quantization

$$P_{m,quad}^- = \frac{L}{2\pi} \sum_{n=1}^{\infty} \frac{1}{n} \left\{ m_b^2 A_{ij}^\dagger(n) A_{ij}(n) + m_f^2 B_{ij}^\dagger(n) B_{ij}(n) \right\}, \quad (29)$$

$$\begin{aligned} P_{m,cubic}^- = & -i \frac{m_fgL}{4\pi^{\frac{3}{2}}} \sum_{n_1, n_2, n_3=1}^{\infty} \left\{ \frac{1}{\sqrt{n_2}} \left[\frac{1}{n_3} + \frac{1}{n_1} \right] \delta_{n_1-n_2+n_3,0} A_{kj}^\dagger(n_2) B_{ki}(n_1) B_{ij}(n_3) \right. \\ & + \frac{1}{\sqrt{n_1}} \left[\frac{1}{n_3} - \frac{1}{n_2} \right] \delta_{n_1-n_2+n_3,0} \left[A_{jk}(n_1) B_{ik}^\dagger(n_2) B_{ij}(n_3) - A_{ki}(n_1) B_{kj}^\dagger(n_2) B_{ij}(n_3) \right] \\ & + \frac{1}{\sqrt{n_1}} \left[\frac{1}{n_3} - \frac{1}{n_2} \right] \delta_{n_1+n_2-n_3,0} \left[A_{kj}^\dagger(n_1) B_{ik}^\dagger(n_2) B_{ij}(n_3) - A_{ik}^\dagger(n_1) B_{kj}^\dagger(n_2) B_{ij}(n_3) \right] \\ & \left. + \frac{1}{\sqrt{n_1}} \left[\frac{1}{n_2} + \frac{1}{n_3} \right] \delta_{n_1-n_2-n_3,0} A_{jk}(n_1) B_{ji}^\dagger(n_2) B_{ik}^\dagger(n_3) \right\}. \end{aligned} \quad (30)$$

4. Results of Supercharge Diagonalization

As we have seen, our procedure preserves supersymmetry manifestly throughout the calculation. Therefore we are naturally led to obtain supersymmetric mass spectra with exactly the same bosonic and fermionic spectra for color singlet states.

If we consider the states with finite values of the discrete momentum K , we have only finitely many physical states to diagonalize the mass matrix. Let us illustrate the procedure for smaller values of the discrete momentum K . In the case of $K = 3$, we find four possible states for bosonic color singlet states

$$\begin{aligned} |1\rangle_b &= \frac{1}{N^{3/2}\sqrt{3}} \text{tr} [A^\dagger(1)A^\dagger(1)A^\dagger(1)] |0\rangle, \\ |2\rangle_b &= \frac{1}{N^{3/2}} \text{tr} [A^\dagger(1)B^\dagger(1)B^\dagger(1)] |0\rangle, \\ |3\rangle_b &= \frac{1}{N} \text{tr} [A^\dagger(2)A^\dagger(1)] |0\rangle, \\ |4\rangle_b &= \frac{1}{N^{1/2}} \text{tr} [B^\dagger(2)B^\dagger(1)] |0\rangle. \end{aligned} \quad (1)$$

and four possible states for fermionic color singlet states

$$\begin{aligned} |1\rangle_f &= \frac{1}{N^{3/2}} \text{tr} [A^\dagger(1)A^\dagger(1)B^\dagger(1)] |0\rangle, \\ |2\rangle_f &= \frac{1}{N^{3/2}\sqrt{3}} \text{tr} [B^\dagger(1)B^\dagger(1)B^\dagger(1)] |0\rangle, \\ |3\rangle_f &= \frac{1}{N} \text{tr} [A^\dagger(2)B^\dagger(1)] |0\rangle, \\ |4\rangle_f &= \frac{1}{N^{1/2}} \text{tr} [B^\dagger(2)A^\dagger(1)] |0\rangle. \end{aligned} \quad (2)$$

Using the matrix B appearing in eq.(16), the mass matrix is given as

$$\frac{\pi}{g^2 N} M^2 = \frac{\pi 2P^+ P^-}{g^2 N} = \frac{\sqrt{2}\pi^2 K}{g^2 N L} Q_2^2, \quad Q_2 = \begin{bmatrix} 0 & B^\dagger \\ B & 0 \end{bmatrix}, \quad (3)$$

$$i \frac{2^{1/4} \pi \sqrt{3}}{g \sqrt{N L}} (B_{ij}) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{9}{2} \\ 0 & -3\sqrt{\frac{3}{2}} & 0 & 0 \\ 0 & \frac{3}{2}\sqrt{3} & 0 & 0 \end{pmatrix}. \quad (4)$$

By diagonalizing this matrix, we obtain mass eigenvalues in units of $g\sqrt{N/\pi}$. We find that two bosonic massless states correspond to two fermionic states through the first supercharge Q_1 as shown in eq.(21)

$$\begin{aligned} |1\rangle_b &\leftrightarrow |1\rangle_f \\ |3\rangle_b &\leftrightarrow \frac{1}{\sqrt{3}} |3\rangle_f + \sqrt{\frac{2}{3}} |4\rangle_f. \end{aligned} \quad (5)$$

We also find that the two bosonic states with mass eigenvalues $81/4$ in units of $g^2 N/\pi$ correspond to two fermionic states with the same eigenvalues which are also mapped by the first supercharge Q_1

$$\begin{aligned} |2\rangle_b &\leftrightarrow |2\rangle_f \\ |4\rangle_b &\leftrightarrow \sqrt{\frac{2}{3}} |3\rangle_f - \frac{1}{\sqrt{3}} |4\rangle_f. \end{aligned} \quad (6)$$

Let us note that the adjoint scalar field alone gives only $|1\rangle_b$ and $|3\rangle_b$ as color singlet states, whereas the adjoint spinor field alone gives $|4\rangle_b$ as bosonic color singlet state and $|2\rangle_f$ as fermionic color singlet state. Therefore each case gives only a quarter of the possible states in our supersymmetric theory.

Similarly for $K = 4$, we find nine possible states for bosonic color singlet states

$$\begin{aligned} |1\rangle_b &= \frac{1}{N^2} \text{tr} \left[A^\dagger(1) A^\dagger(1) A^\dagger(1) A^\dagger(1) \right] |0\rangle \\ |2\rangle_b &= \frac{1}{N^2} \text{tr} \left[A^\dagger(1) A^\dagger(1) B^\dagger(1) B^\dagger(1) \right] |0\rangle \\ |3\rangle_b &= \frac{1}{N^{3/2}} \text{tr} \left[A^\dagger(2) A^\dagger(1) A^\dagger(1) \right] |0\rangle \\ |4\rangle_b &= \frac{1}{N^{3/2}} \text{tr} \left[A^\dagger(2) B^\dagger(1) B^\dagger(1) \right] |0\rangle \\ |5\rangle_b &= \frac{1}{N^{3/2}} \text{tr} \left[A^\dagger(1) B^\dagger(1) B^\dagger(2) \right] |0\rangle \\ |6\rangle_b &= \frac{1}{N^{3/2}} \text{tr} \left[A^\dagger(1) B^\dagger(2) B^\dagger(1) \right] |0\rangle \\ |7\rangle_b &= \frac{1}{N} \text{tr} \left[A^\dagger(3) A^\dagger(1) \right] |0\rangle \\ |8\rangle_b &= \frac{1}{N\sqrt{2}} \text{tr} \left[A^\dagger(2) A^\dagger(2) \right] |0\rangle \\ |9\rangle_b &= \frac{1}{N} \text{tr} \left[B^\dagger(3) B^\dagger(1) \right] |0\rangle, \end{aligned} \quad (7)$$

and nine possible states for fermionic color singlet states

$$\begin{aligned} |1\rangle_f &= \frac{1}{N^2} \text{tr} \left[A^\dagger(1) A^\dagger(1) A^\dagger(1) B^\dagger(1) \right] |0\rangle \\ |2\rangle_f &= \frac{1}{N^2} \text{tr} \left[A^\dagger(1) B^\dagger(1) B^\dagger(1) B^\dagger(1) \right] |0\rangle \\ |3\rangle_f &= \frac{1}{N^{3/2}} \text{tr} \left[A^\dagger(2) A^\dagger(1) B^\dagger(1) \right] |0\rangle \\ |4\rangle_f &= \frac{1}{N^{3/2}} \text{tr} \left[A^\dagger(2) B^\dagger(1) A^\dagger(1) \right] |0\rangle \\ |5\rangle_f &= \frac{1}{N^{3/2}} \text{tr} \left[B^\dagger(2) A^\dagger(1) A^\dagger(1) \right] |0\rangle \\ |6\rangle_f &= \frac{1}{N^{3/2}} \text{tr} \left[B^\dagger(2) B^\dagger(1) B^\dagger(1) \right] |0\rangle \\ |7\rangle_f &= \frac{1}{N} \text{tr} \left[A^\dagger(3) B^\dagger(1) \right] |0\rangle \\ |8\rangle_f &= \frac{1}{N} \text{tr} \left[A^\dagger(2) B^\dagger(2) \right] |0\rangle \\ |9\rangle_f &= \frac{1}{N} \text{tr} \left[B^\dagger(3) A^\dagger(1) \right] |0\rangle. \end{aligned} \quad (8)$$

The matrix B appearing in the supercharge Q_2 in eq.(16) is given by

$$i \frac{2^{5/4} \pi}{g \sqrt{NL}} (B_{ij}) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & -3 & 0 & 0 & 0 \\ 0 & -\frac{3}{\sqrt{2}} & 0 & 0 & 0 & 0 & -\frac{5}{\sqrt{6}} & 3 & \frac{1}{3\sqrt{2}} \\ 0 & -\frac{3}{\sqrt{2}} & 0 & 0 & 0 & 0 & \frac{5}{\sqrt{6}} & -3 & -\frac{1}{3\sqrt{2}} \\ 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{14}{3} \\ 0 & 0 & 0 & -5\sqrt{\frac{2}{3}} & -\frac{2}{\sqrt{3}} & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & -\frac{3}{\sqrt{2}} & -\frac{3}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{7}{3} & \frac{7}{3} & 0 & 0 & 0 \end{pmatrix}. \quad (9)$$

From the diagonalization of the matrix for bosonic color singlet states, we find four different mass eigenvalues 0, 18, and $(1302 \pm 42\sqrt{13})/54$. All massive states have degeneracy two, whereas there are three massless states. We find exactly the same spectra for fermionic color singlet states.

We have explicitly constructed bosonic and fermionic color singlet states for higher values of the cut-off momentum K up to $K = 11$. We find the number of bosonic color singlet states for $K = 5, 6, 7, 8, 9, 10$, and 11 to be 24, 61, 156, 409, 1096, 2953, and 8052 respectively. The number of fermionic color singlet states is exactly the same as the corresponding bosonic one with the same K .

After evaluating the supercharge for these subspace up to $K = 8$, we diagonalize the supercharge exactly to obtain the mass eigenvalues. In Fig.1 we plot the accumulated number of bosonic color singlet bound states as a function of mass squared in units of $g^2 N / \pi$. We can see that the number of states is approaching to a limiting value at least for smaller values of M^2 . The present tendency seems to suggest that the density of states is increasing rapidly as the mass squared increases. This behavior is in qualitative agreement with the previous results for the adjoint scalar or adjoint spinor matter constituents in nonsupersymmetric gauge theories [7]. Namely the density of states showed an exponential increase as mass squared increases in accordance with the closed string interpretation. The fermionic color singlet bound states show the same behavior.

In Fig.2, we plot the mass squared of bosonic color singlet bound states in units of $g^2 N / \pi$ as a function of the average number of constituents for the case of $K = 5$. We have also obtained a similar plot of the fermionic color singlet bound states which turns out to be indistinguishable from the bosonic one. Since we find the exact correspondence, we shall display only the bosonic spectra. In Figs.3, 4, and 5 we plot the mass squared in units of $g^2 N / \pi$ as a function of the average number of

constituents for the case of $K = 6, 7$, and 8 respectively. It is interesting to see that the average number of constituents increases as mass squared increases.

We find that there are a number of massless states. Empirically we find that there are $K - 1$ bosonic and fermionic massless states for the momentum cut-off K . It is easy to understand some of the massless states. For instance, for each K there is one massless bosonic state with K bosonic oscillators of the lowest level $A^\dagger(1)$ acting on the vacuum. There is also one massless bosonic state with one bosonic oscillator $A^\dagger(2)$ of level two and $K - 2$ bosonic oscillators of the lowest level $A^\dagger(1)$ acting on the vacuum. Both these states become massless at arbitrary K because of the principal value prescription for the infrared regularization of the supercharge.

The bound state equations for adjoint scalar or spinor constituents are infinitely coupled even in the large N limit [7]. To compare with the case of constituents in the fundamental representation, it is instructive to work out a truncation to a two constituents subspace. The two-body bound state equation becomes analogous to but is somewhat different from the 'tHooft equation [4] extended to the boson-boson bound state case [14] and the boson-fermion bound state case [15], as given in Appendix B. Unfortunately, our results in Figs.2–5 suggest that the two-body truncation does not seem to give an adequate approximation even for states with low excitations.

To explore the effects of supersymmetry breaking mass terms, we diagonalize the mass matrix exactly with equal mass $m = m_b = m_f$ for scalar and spinor constituents. The explicit form of the mass matrix for $K = 3$ and $K = 4$ are given in Appendix C. As an illustration, we plot the mass squared of bosonic color singlet bound states for $K = 4$ as a function of the constituent mass both in unit of $g\sqrt{N}/\pi$ in Fig.6. Similarly Fig.7 shows the fermionic bound state. We observe that the mass spectra of bosonic bound states and fermionic ones differ as constituent mass increases even though we have given identical masses for both bosonic and fermionic constituents. This is because they are supersymmetric partners of gauge boson which has to be massless. It is interesting to see that the vanishing mass of the gauge boson demands massless scalars and spinors even though the gauge boson does not have dynamical degree of freedom.

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Appendix A. Superfield and Supertransformation

Here we construct the action of supersymmetric Yang-Mills theory in $1 + 1$ dimensions by using the superfield formalism.

The spinor superfield V_α corresponds to the vector multiplet

$$V_\alpha(x, \theta) = \xi_\alpha(x) - \frac{i}{2}(\gamma_5 \gamma_\mu \theta)_\alpha \frac{A^\mu(x)}{g} + \frac{1}{2}\theta_\alpha \phi(x) - \frac{1}{2}N(x)(\gamma_5 \theta)_\alpha - \frac{1}{2}\bar{\theta}\theta\sqrt{2}\Psi_\alpha(x), \quad (1)$$

where θ is a two-component Majorana Grassmann spinor, ξ_α, Ψ_α are Majorana spinors, A^μ is a vector field, ψ and N are scalar fields. Spinor indices and spacetime indices are denoted by α and μ respectively. The infinitesimal gauge transformation on V_α is defined by

$$\delta_{gauge} V_\alpha = -(\gamma_5 D)_\alpha S - i2g[S, V_\alpha] = -(\gamma_5 \nabla)_\alpha S, \quad (2)$$

$$(\nabla)_\alpha S \equiv D_\alpha S - i2g[(\gamma_5 V)_\alpha, S], \quad D_\alpha = -\frac{\partial}{\partial \theta^\alpha} + i(\gamma^\mu \theta)_\alpha \frac{\partial}{\partial x^\mu}, \quad (3)$$

where D_α is the supercovariant derivative and ∇ is the super- as well as gauge-covariant derivative. The transformation parameter S is a scalar superfield:

$$S(x, \theta) = \Lambda(x) - \bar{\theta}\lambda(x) - \frac{1}{2}\bar{\theta}\theta F(x), \quad (4)$$

where Λ, F are scalar fields, and λ is a two-component Majorana spinor field.

Let us define the quantity \tilde{G} which transforms covariantly under the gauge transformation (2)

$$\tilde{G} = \bar{D}V + i2g\bar{V}\gamma_5 V, \quad \delta_{gauge} \tilde{G} = -i2g[S, \tilde{G}]. \quad (5)$$

Using $\int d^2\theta \frac{1}{2}\bar{\theta}\theta = -1$, the action of supersymmetric Yang-Mills theory is given by the covariant derivative of \tilde{G}

$$\nabla_\alpha \tilde{G} = D_\alpha \tilde{G} - i2g[\gamma_5 V, \tilde{G}], \quad \delta_{gauge} \nabla_\alpha \tilde{G} = -i2g[S, \nabla_\alpha \tilde{G}]. \quad (6)$$

$$S = \int d^2\theta d^2x \left[-\frac{1}{4} \text{tr} \left(\bar{\nabla}_\alpha \tilde{G} \nabla_\alpha \tilde{G} \right) \right]. \quad (7)$$

Let us decompose the gauge transformation (2) in component fields

$$\begin{aligned} \delta_{gauge} \xi_\alpha &= -(\gamma_5 \lambda)_\alpha - i2g[\Lambda, \xi_\alpha], \\ \delta_{gauge} A_\mu / g &= -i2[\Lambda, A_\mu] - 2g\{\bar{\lambda}, \gamma_5 \gamma_\mu \xi\} + 2\partial_\mu \Lambda, \\ \delta_{gauge} \phi &= -i2g[\Lambda, \phi] - i2g\{\bar{\lambda}, \xi\}, \\ \delta_{gauge} N &= 2F - i2g[\Lambda, N] + i2g\{\bar{\lambda}, \gamma_5 \xi\}, \\ \delta_{gauge} \sqrt{2}\Psi &= i\gamma_5 \gamma_\mu \partial^\mu \lambda - i2g[\Lambda, \sqrt{2}\Psi] + [\gamma_5 \gamma^\mu \lambda, A_\mu] \\ &\quad + ig[\lambda, \phi] - ig[\gamma_5 \lambda, N] - i2g[F, \xi]. \end{aligned} \quad (8)$$

We choose the Wess-Zumino gauge $\xi_\alpha = N = 0$ by using the gauge freedom λ and F , and we find the remaining gauge transformation with the parameter Λ

$$\begin{aligned} \delta_{gauge} A_\mu &= -i2g[\Lambda, A_\mu] + 2g\partial_\mu \Lambda, \\ \delta_{gauge} \phi &= -i2g[\Lambda, \phi], \quad \delta_{gauge} \sqrt{2}\Psi = -i2g[\Lambda, \sqrt{2}\Psi]. \end{aligned} \quad (9)$$

Next we consider the supertransformation. The superfield V transforms as

$$\delta_{super} V = -i\bar{\epsilon} Q V = \bar{\epsilon} \left[-\frac{\partial}{\partial \theta} - i\gamma^\mu \theta \partial_\mu \right] V, \quad (10)$$

where Q_α is the supercharge acting on superfields and ϵ is an infinitesimal two-component Majorana spinor. In terms of components, it becomes

$$\begin{aligned} \delta_{super} \xi_\alpha &= \frac{i}{2} (\gamma_5 \gamma^\mu \epsilon)_\alpha \frac{A_\mu}{g} - \frac{1}{2} \epsilon_\alpha \phi + \frac{1}{2} (\gamma_5 \epsilon)_\alpha N, \\ \delta_{super} A_\mu &= ig\bar{\epsilon} \gamma_5 \left(\gamma_\mu \sqrt{2}\Psi + i\gamma^\nu \gamma_\mu \partial_\nu \xi \right), \\ \delta_{super} \phi &= -\bar{\epsilon} \left(\sqrt{2}\Psi - i\gamma^\nu \partial_\nu \xi \right), \\ \delta_{super} N &= \bar{\epsilon} \gamma_5 \left(\sqrt{2}\Psi + i\gamma^\nu \partial_\nu \xi \right), \\ \delta_{super} \Psi_\alpha &= \frac{1}{2} (\gamma_5 \gamma^\nu \gamma^\mu \epsilon)_\alpha \frac{\partial_\mu A_\nu}{g} + \frac{i}{2} (\gamma^\mu \epsilon)_\alpha \partial_\mu \phi - \frac{i}{2} (\gamma_5 \gamma^\mu \epsilon)_\alpha \partial_\mu N. \end{aligned} \quad (11)$$

Note that the Wess-Zumino gauge condition $\xi = N = 0$ is violated by the supertransformation. We therefore need to make compensating gauge transformation to maintain the Wess-Zumino gauge condition

$$\delta_{super} \xi + \delta_{gauge} \xi = 0, \quad \delta_{super} N + \delta_{gauge} N = 0, \quad (12)$$

at the Wess-Zumino gauge fixing slice $\xi = N = 0$. By choosing the compensating gauge transformation as

$$\lambda = \frac{i}{2} \gamma^\mu \epsilon \frac{A_\mu}{g} - \frac{1}{2} \gamma_5 \epsilon \phi, \quad F = -\frac{1}{2} \bar{\epsilon} \gamma_5 \sqrt{2}\Psi. \quad (13)$$

we obtain the modified supertransformation $\tilde{\delta}_{super} \equiv \delta_{super} + \delta_{gauge}$ in the Wess-Zumino gauge as shown in eq.(3). In the Wess-Zumino gauge, \tilde{G} becomes

$$\tilde{G} = \phi + \bar{\theta}\sqrt{2}\Psi + \frac{1}{2}\bar{\theta}\theta\frac{1}{2g}\epsilon^{\mu\nu}F_{\mu\nu}, \quad (14)$$

where $\epsilon^{01} = -\epsilon_{01} = 1$. Substituting (14) into (7), we obtain the action (1) of the two dimensional supersymmetric Yang-Mills theory. Since the light-cone gauge condition $A_- = 0$ is violated by the supertransformation (3), we need to define a modified supertransformation $\tilde{\tilde{\delta}}_{super} \equiv \tilde{\delta}_{super} + \delta_{gauge}$ by adding another compensating gauge transformation to make $\tilde{\tilde{\delta}}_{super}A_- = 0$. We find the supertransformation for the dynamical variables in the light-cone gauge as

$$\tilde{\tilde{\delta}}_{super}\phi = i2^{1/4}(\epsilon_1\chi - \epsilon_2\psi) + 2^{3/4}g\epsilon_1\left[\frac{1}{\partial_-}\psi, \phi\right] \quad (15)$$

$$\tilde{\tilde{\delta}}_{super}\psi = -\frac{1}{2^{1/4}g}\epsilon_1\partial_-A_+ + 2^{1/4}\epsilon_2\partial_-\phi + 2^{3/4}g\epsilon_1\left[\frac{1}{\partial_-}\psi, \psi\right] \quad (16)$$

Appendix B. Tow-body Truncation of Bound State Equations

Here we summarize the bound state equations in the truncated subspace of two constituents only. Bosonic bound states consist of two bosonic constituents wave function ϕ_{bb} and two fermionic one ϕ_{ff}

$$\begin{aligned} |\Phi(P^+)\rangle_b = \int_0^{P^+} dk_1 dk_2 \delta(k_1 + k_2 - P^+) & \left\{ \phi_{bb}(k_1, k_2) \frac{1}{N} \text{tr}[a^\dagger(k_1), a^\dagger(k_2)] \right. \\ & \left. + \phi_{ff}(k_1, k_2) \frac{1}{N} \text{tr}[b^\dagger(k_1), b^\dagger(k_2)] \right\} |0\rangle, \end{aligned} \quad (1)$$

$$\phi_{bb}(k, P^+ - k) = \phi_{bb}(P^+ - k, k), \quad \phi_{ff}(k, P^+ - k) = -\phi_{ff}(P^+ - k, k). \quad (2)$$

We define $C_j = C + \frac{2\pi m_j}{g^2 N}$, ($j = b, f$), using the quadratic term in the Hamiltonian $C(k)$ defined in eq.(29). We obtain a coupled bound state equation for bosonic

bound states using $x \equiv k/P^+$, $y \equiv l/P^+$

$$\begin{aligned}
M^2 \phi_{bb}(k, P^+ - k) &= \frac{g^2 N}{2\pi} \left[\frac{C_b(k)}{x} + \frac{C_b(P^+ - k)}{1-x} \right] \phi_{bb}(k, P^+ - k) \\
&- \frac{g^2 N}{2\pi} \int_0^1 \frac{dy}{\sqrt{x(1-x)y(1-y)}} \frac{(x+y)(2-x-y)}{(x-y)^2} \phi_{bb}(l, P^+ - l) \\
&+ \frac{g^2 N}{2\pi} \int_0^1 \frac{dy}{(y-x)\sqrt{x(1-x)}} \phi_{ff}(l, P^+ - l),
\end{aligned} \tag{3}$$

$$\begin{aligned}
M^2 \phi_{ff}(k, P^+ - k) &= \frac{g^2 N}{2\pi} \left[\frac{C_f(k)}{x} + \frac{C_f(P^+ - k)}{1-x} \right] \phi_{ff}(k, P^+ - k) \\
&- \frac{2g^2 N}{\pi} \int_0^1 \frac{dy}{(x-y)^2} \phi_{ff}(l, P^+ - l) + \frac{g^2 N}{2\pi} \int_0^1 dy \frac{\phi_{bb}(l, P^+ - l)}{(x-y)\sqrt{y(1-y)}}.
\end{aligned} \tag{4}$$

Similarly we find a bound state equation for fermionic ones

$$\left| \Phi(P^+) \right\rangle_f = \int_0^{P^+} dk_1 dk_2 \delta(k_1 + k_2 - P^+) \phi_{bf}(k_1, k_2) \frac{1}{N} \text{tr}[a^\dagger(k_1), b^\dagger(k_2)] |0\rangle, \tag{5}$$

$$\begin{aligned}
M^2 \phi_{bf}(k, P^+ - k) &= \frac{g^2 N}{2\pi} \left[\frac{C_b(k)}{x} + \frac{C_f(P^+ - k)}{1-x} \right] \phi_{bf}(k, P^+ - k) \\
&- \frac{g^2 N}{2\pi} \int_0^1 \frac{dy}{(1-x-y)\sqrt{xy}} \phi_{bf}(l, P^+ - l) - \frac{g^2 N}{\pi} \int_0^1 dy \frac{x+y}{(x-y)^2 \sqrt{xy}} \phi_{bf}(l, P^+ - l).
\end{aligned} \tag{6}$$

Appendix C. Mass Matrix with Massive Constituents

Here we display the bound state mass matrices for massive constituents. Introducing mass squared parameters of constituents in unit of $g^2 N/\pi$

$$x = \frac{\pi m_b^2}{g^2 N}, \quad y = \frac{\pi m_f^2}{g^2 N} \tag{1}$$

we find the mass squared matrix in unit of $g^2 N/\pi$ for $K = 3$ bosonic bound states defined in eq.(1)

$$\frac{M^2 \pi}{g^2 N} = \begin{pmatrix} 9x & 0 & 0 & 0 \\ 0 & \frac{81}{4} + 3x + 6y & -3i\sqrt{\frac{y}{2}} & -\frac{3i}{2}\sqrt{y} \\ 0 & 3i\sqrt{\frac{y}{2}} & \frac{9}{2}x & 0 \\ 0 & \frac{3i}{2}\sqrt{y} & 0 & \frac{81}{4} + \frac{9}{2}y \end{pmatrix} \tag{2}$$

For $K = 3$ fermionic bound states defined in eq.(2), we find

$$\frac{M^2 \pi}{g^2 N} = \begin{pmatrix} 3x + 6y & 0 & 0 & 0 \\ 0 & \frac{81}{4} + 9y & -3i\sqrt{\frac{3y}{2}} & 0 \\ 0 & 3i\sqrt{\frac{3y}{2}} & \frac{27}{2} + \frac{3}{2}x + 3y & -\frac{27}{2\sqrt{2}} \\ 0 & 0 & -\frac{27}{2\sqrt{2}} & \frac{27}{4} + 3x + \frac{3}{2}y \end{pmatrix} \quad (3)$$

For $K = 4$ bound states, we decompose the mass matrix as

$$\frac{\pi}{g^2 N} M^2 = \frac{\sqrt{2}\pi^2 K}{g^2 N L} Q_2^2 + T_x + T_y \quad (4)$$

For bosonic bound states, the first supersymmetric term Q_2^2 reduces to $B^\dagger B$ in terms of the matrix B in eq.(9), and the second and third terms are given by

$$T_x = \begin{pmatrix} 16x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8x & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10x & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2x & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4x & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4x & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{16}{3}x & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4x & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (5)$$

$$T_y = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8y & -2i\sqrt{2y} & 0 & i\sqrt{y} & -i\sqrt{y} & 0 & 0 & 0 \\ 0 & 2i\sqrt{2y} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 8y & 0 & 0 & 0 & -4i\sqrt{y} & -\frac{4i}{3}\sqrt{2y} \\ 0 & -i\sqrt{y} & 0 & 0 & 6y & 0 & -i\sqrt{3y} & 0 & -\frac{i}{3}\sqrt{y} \\ 0 & i\sqrt{y} & 0 & 0 & 0 & 6y & -i\sqrt{3y} & 0 & -\frac{i}{3}\sqrt{y} \\ 0 & 0 & 0 & 0 & i\sqrt{3y} & i\sqrt{3y} & 0 & 0 & 0 \\ 0 & 0 & 0 & 4i\sqrt{y} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{4i}{3}\sqrt{2y} & \frac{i}{3}\sqrt{y} & \frac{i}{3}\sqrt{y} & 0 & 0 & \frac{16}{3}y \end{pmatrix} \quad (6)$$

For fermionic bound states, the Q_2^2 reduces to BB^\dagger in terms of the matrix B in

eq.(9), and the second and third terms are given by

$$T_x = \begin{pmatrix} 12x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4x & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 6x & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6x & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8x & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{4}{3}x & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2x & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4x \end{pmatrix} \quad (7)$$

$$T_y = \begin{pmatrix} 4y & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 12y & -2i\sqrt{2y} & -2i\sqrt{2y} & 0 & 0 & 0 & 0 & 0 \\ 0 & 2i\sqrt{2y} & 4y & 0 & 0 & 0 & 0 & -i\sqrt{y} & \frac{2i}{3}\sqrt{2y} \\ 0 & 2i\sqrt{2y} & 0 & 4y & 0 & 0 & 0 & i\sqrt{y} & -\frac{2i}{3}\sqrt{2y} \\ 0 & 0 & 0 & 0 & 2y & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10y & -2i\sqrt{3y} & -2i\sqrt{2y} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2i\sqrt{3y} & 4y & 0 & 0 \\ 0 & 0 & i\sqrt{y} & -i\sqrt{y} & 0 & 2i\sqrt{2y} & 0 & 2y & 0 \\ 0 & 0 & -\frac{2i}{3}\sqrt{2y} & \frac{2i}{3}\sqrt{2y} & 0 & 0 & 0 & 0 & \frac{4}{3}y \end{pmatrix} \quad (8)$$

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Figure 1: The accumulated number of bound states as a function of mass squared for $K = 4, 5, 6, 7, 8$; there is no difference in behavior between bosonic and fermionic state.

k5fig.eps

Figure 2: Mass squared of bosonic bound states for $K = 5$ as a function of the average number of constituents; M^2 are measured in units of $g^2 N/\pi$

Figure captions

Fig. 1 The accumulated number of bound states as a function of mass squared for $K = 4, 5, 6, 7, 8$; there is no difference in behavior between bosonic and fermionic state.

Fig. 2 Mass squared of bosonic bound states for $K = 5$ as a function of the average number of constituents; M^2 are measured in units of $g^2 N/\pi$.

Fig. 3 Mass squared of $K = 6$ bosonic bound states as a function of the average number of constituents; M^2 are measured in units of $g^2 N/\pi$.

Fig. 4 Mass squared of $K = 7$ bosonic bound states as a function of the average number of constituents; M^2 are measured in units of $g^2 N/\pi$.

Fig. 5 Mass squared of $K = 8$ bosonic bound states as a function of the average number of constituents; M^2 are measured in units of $g^2 N/\pi$.

Fig. 6 Mass squared of $K = 4$ bosonic bound states as a function of the constituent mass squared; both are measured in units of $g^2 N/\pi$.

Fig. 7 Mass squared of $K = 4$ fermionic bound states as a function of the constituent mass squared; both are measured in units of $g^2 N/\pi$.

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Figure 3: Mass squared of $K = 6$ bosonic bound states as a function of the average number of constituents; M^2 are measured in units of $g^2 N/\pi$

k7fig.eps

Figure 4: Mass squared of $K = 7$ bosonic bound states as a function of the average number of constituents; M^2 are measured in units of $g^2 N/\pi$

k8bfig.eps

Figure 5: Mass squared of bosonic bound states for $K = 8$ as a function of the average number of constituents; M^2 are measured in units of $\frac{g^2 N}{\pi}$.

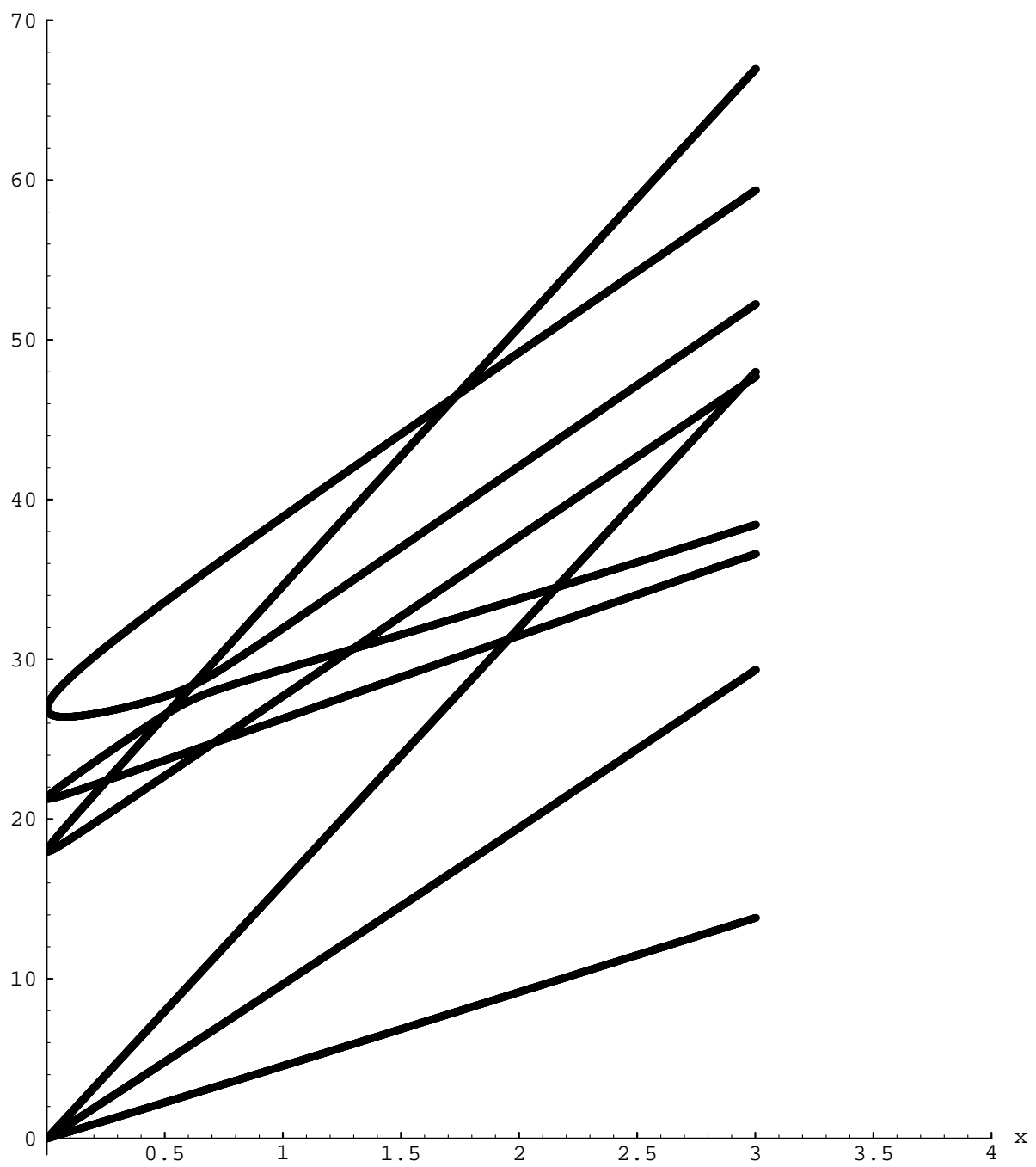
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Figure 6: Mass squared of $K = 4$ bosonic bound states as a function of the constituent mass squared; both are measured in units of $g^2 N/\pi$.

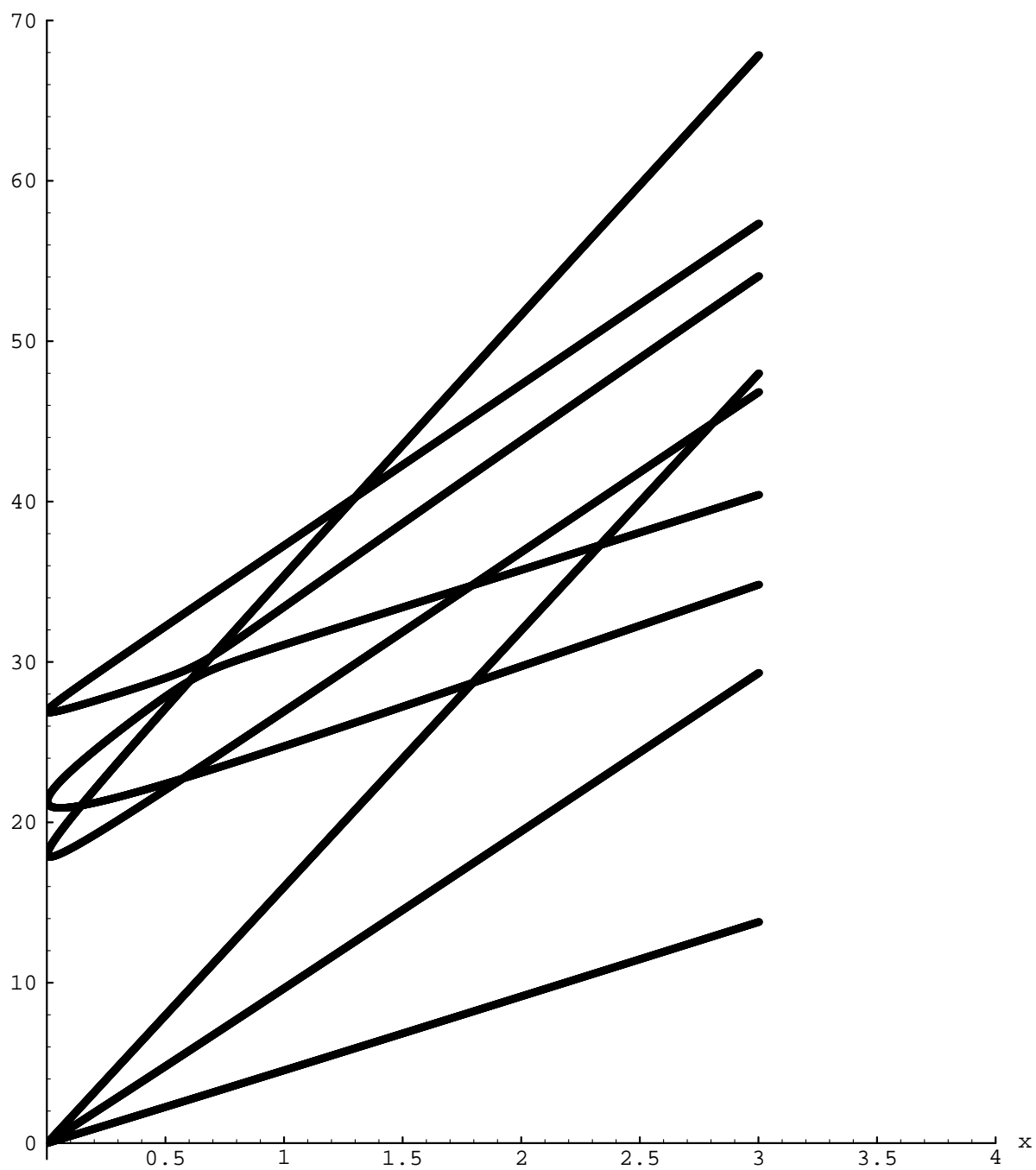
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Figure 7: Mass squared of $K = 4$ fermionic bound states as a function of the constituent mass squared; both are measured in units of $g^2 N/\pi$.

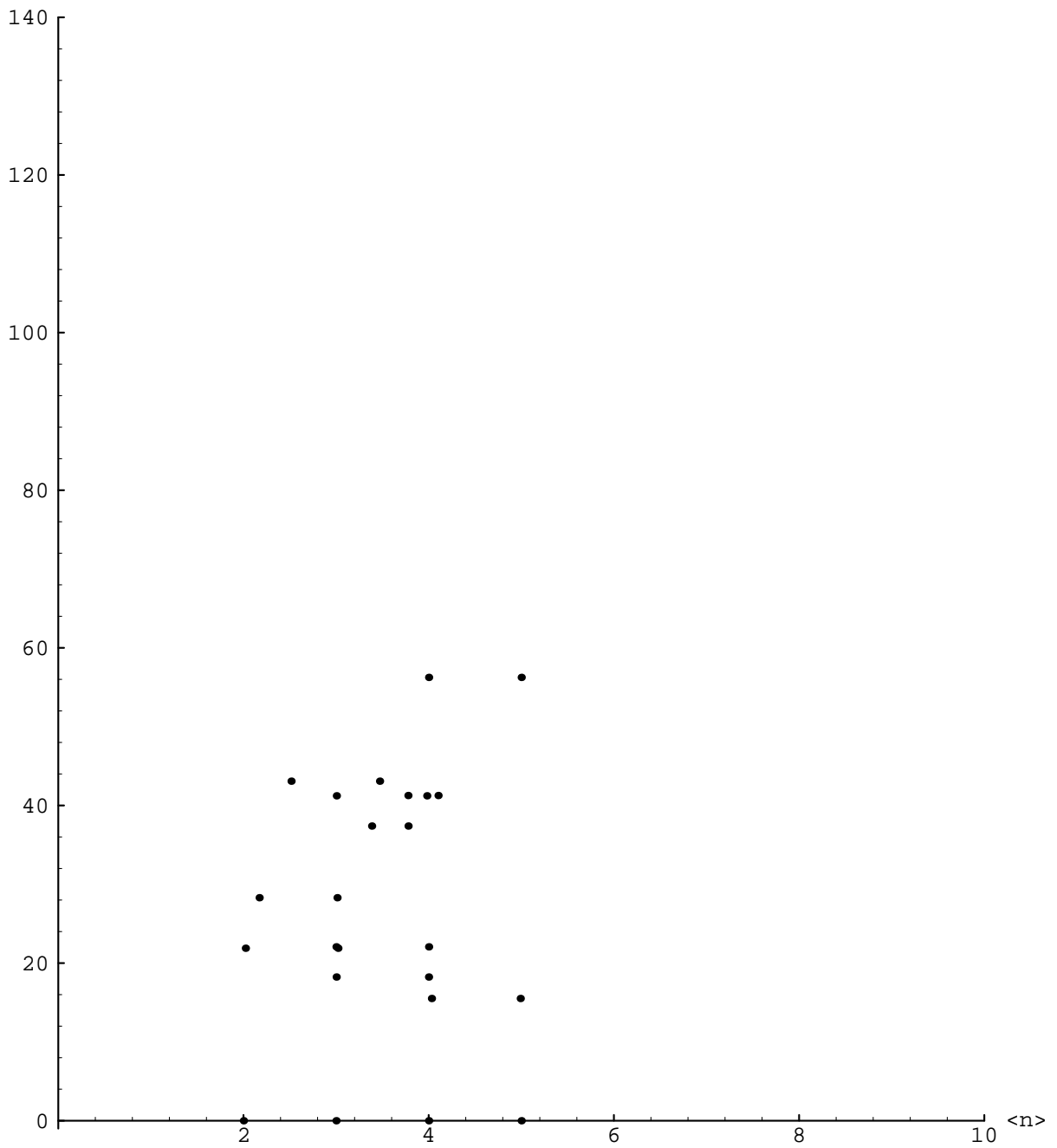
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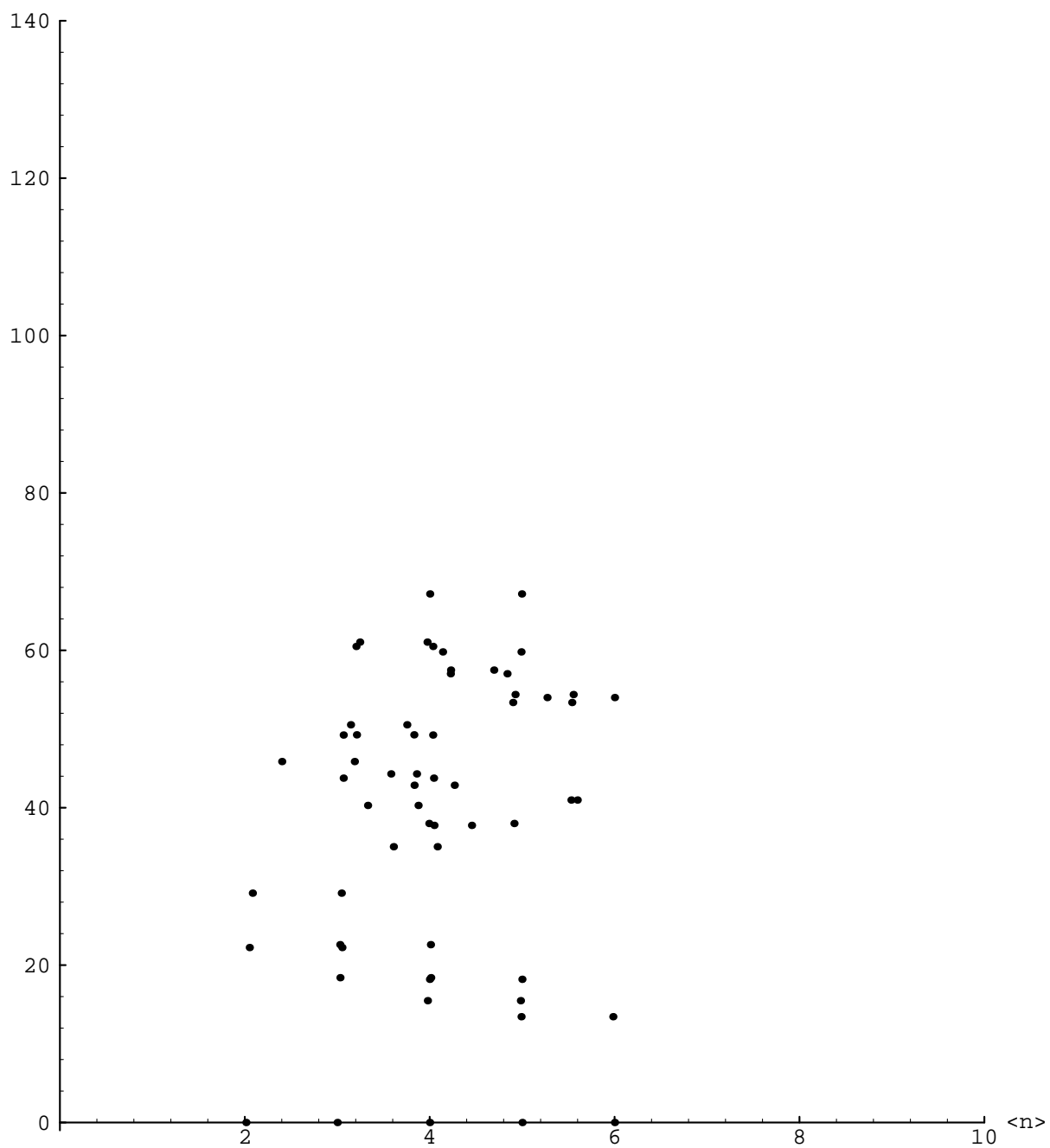
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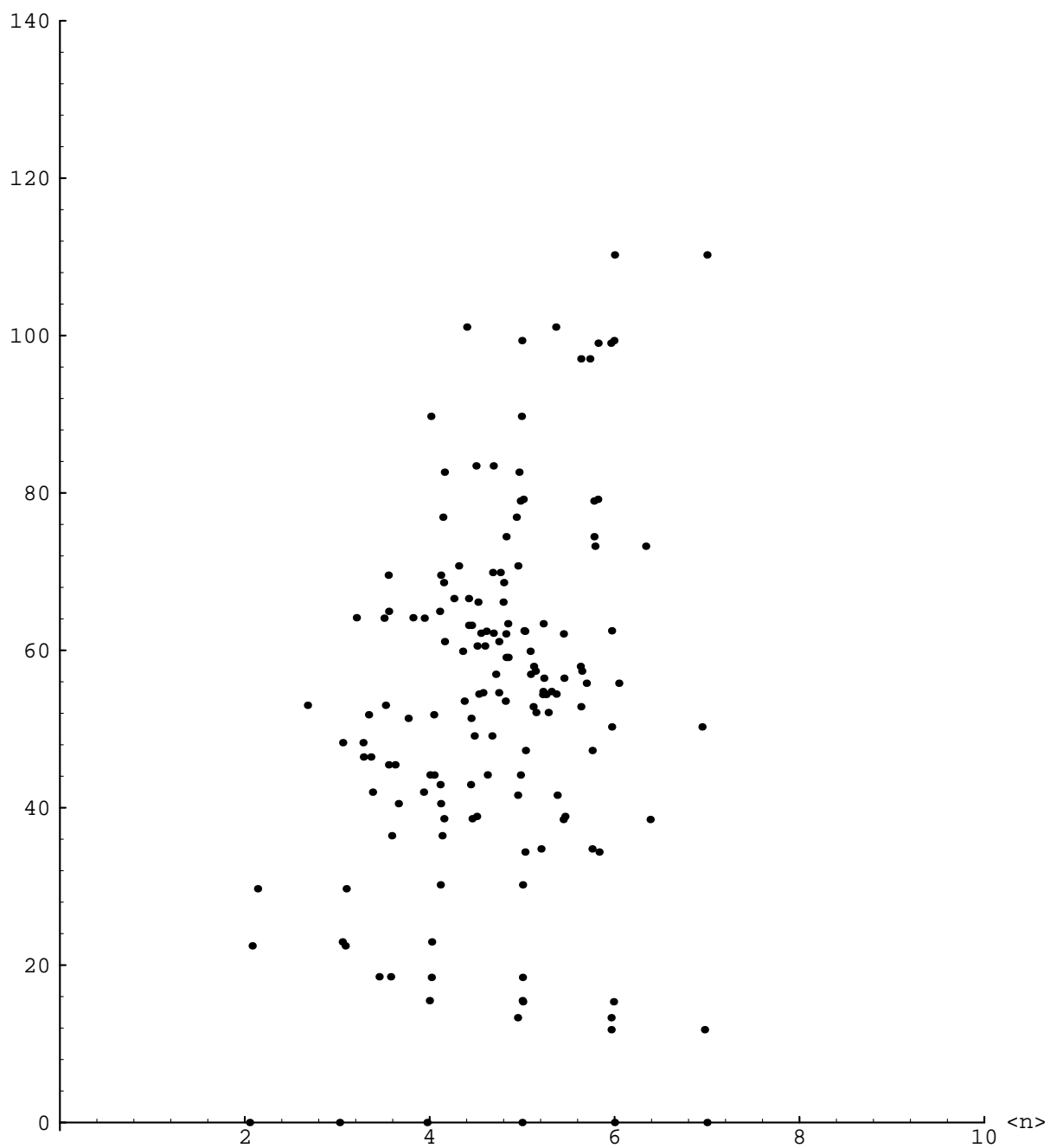
$M^2/(g^2 N/\pi)$



$M^2/(g^2 N/\pi)$



$M^2/(g^2 N/\pi)$



$M^2/(g^2 N/\pi)$

